

CHAPTER THIRTEEN

SURDS

When a square roots sign is squared or raised to the second power, the square root sign disappears.

$$(1) \quad (\sqrt{2})^2 = 2 \qquad (2)(\sqrt{3})^2 = 3$$

$$(3) \quad (\sqrt{a})^2 = a \qquad (4)(\sqrt{b})^2 = b$$

$$(5) \quad 2(\sqrt{3})^2 = 2(3) = 6$$

$$(6) \quad 5(\sqrt{2})^2 = 5(2) = 10$$

$$(7) \quad 2(\sqrt{a})^2 = 2(a) = 2a$$

$$(8) \quad (2\sqrt{3})^2 = 2^2 (\sqrt{3})^2 = 4(3) = 12$$

$$(9) \quad (5\sqrt{2})^2 = 5^2 (\sqrt{2})^2 = 25(2) = 50$$

$$(10) \quad (a\sqrt{b})^2 = a^2 (\sqrt{b})^2 = a^2(b) = a^2b$$

The Perfect Squares:

The Perfect squares are:

$$(1) \quad 2 \times 2 = 4 \qquad (2) 3 \times 3 = 9$$

$$(3) \quad 4 \times 4 = 16 \qquad (4) 5 \times 5 = 25$$

$$(5) \quad 6 \times 6 = 36 \qquad (6) 7 \times 7 = 49$$

$$(7) \quad 8 \times 8 = 64 \qquad (8) 9 \times 9 = 81$$

$$(9) \quad 10 \times 10 = 100$$

-- In surd manipulation, a number which is a multiple of a perfect square must be converted into the multiple of that perfect square

Examples:

$$1. \quad \sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2 \times \sqrt{2} = 2\sqrt{2}.$$

$$2. \quad \sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2 \times \sqrt{3} = 2\sqrt{3}.$$

$$3. \quad \sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} \\ = 4 \times \sqrt{2} = 4\sqrt{2}$$

$$4. \quad \sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$$

$$5. \quad \sqrt{69} = \sqrt{9 \times 7} = \sqrt{9} \times \sqrt{7} = 3\sqrt{7}$$

$$6. \quad \sqrt{80} = \sqrt{16 \times 5} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$$

$$(7) \quad \sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$$

$$(8) \quad \sqrt{147} = \sqrt{49 \times 3} = \sqrt{49} \times \sqrt{3} = 7\sqrt{3}$$

$$(9) \quad \sqrt{125} = \sqrt{25 \times 5} = \sqrt{25} \times \sqrt{5} = 5\sqrt{5}$$

How to determine whether a number is a multiple of a perfect square:

-- The perfect squares 4, 9, 16, 25, 36, 49, 64, 81 and 100 are what we shall make use of.

-- When a number is given and we want to know whether it is a multiple of a perfect square, we start dividing the numbers with the perfect squares in turn, starting with the highest which is 100.

-- If we get an answer which is a whole number but not a decimal, then that particular number is a multiple of a perfect square.

-- For example if we are given $\sqrt{448}$ and we want to know whether or not we can break it down, we first divide 448 by 100 which gives us 4.48.

-- Since this is a decimal, then we try the next perfect square which is 81.

-- Dividing 448 by 81 gives us 5.5 which is also a decimal. We then try the next perfect square which is 64.

-- Dividing 448 by 64 gives us 7, which is a whole number.

$$- \quad \text{This implies that } \sqrt{448} \\ = \sqrt{64 \times 7} = \sqrt{64} \times \sqrt{7} = 8\sqrt{7}.$$

Example (2) Let us now determine whether $\sqrt{294}$ can be simplified or broken down.

- Dividing 294 by 100 gives us 2.94 which is a decimal.

- Dividing 294 by 81 gives us 3.6 which is a decimal.
- Dividing 294 by 64 gives us 4.59 which is also a decimal.
- Dividing 294 by 49 gives us 6 which is a whole number
- This implies that $\sqrt{294}$
 $= \sqrt{49 \times 6} = \sqrt{49} \times \sqrt{6} = 7\sqrt{6}$.

Examples (3) Now let us determine whether $\sqrt{150}$ can be simplified or broken down.

- Dividing 150 by 100 gives us 1.50 which is a decimal
- Dividing 150 by 81 gives us 1.85 which is a decimal.
- Dividing 150 by 49 gives us 3.06 which is a decimal.
- Dividing 150 by 36 gives us 4.2 which is a decimal.
- Dividing 150 by 25 gives us 6 which is a whole number.
- This implies that $\sqrt{150}$
 $= \sqrt{25 \times 6} = \sqrt{25} \times \sqrt{6} = 5\sqrt{6}$.

NB: If we divide a given number by all the perfect Squares we are supposed to use, and in each case get a decimal, then that number cannot be simplified or broken down.

For example, assume that we want to know whether or not $\sqrt{271}$ can be simplified or broken down.

- Dividing 271 by 100 gives us 2.71 which is a decimal.
- Dividing 271 by 81 gives us 3.34 which is a decimal
- Dividing 271 by 64 gives us 4.2 which is decimal.
- Dividing 271 by 49 gives us 5.5 which is decimal.
- Dividing 271 by 36 gives us 7.5 which is a decimal.
- Dividing 271 by 25 gives us 10.8 which is decimal.
- Dividing 271 by 16 gives us 16.9 which is decimal.

-- Dividing 271 by 9 gives us 30.1 which is a decimal.

-- Dividing 271 by 4 gives us 67.7 which is also a decimal.

Since when all the perfect squares concerned, when used to divide the given number gave us decimals as our answer, then $\sqrt{271}$ cannot be simplified and should be left as $\sqrt{271}$.

Addition of surds:

-- In surd addition, we can only add if the numbers under the square root signs are the same.

-- If they are not the same, then nothing can be done.

Examples

$$(1) \quad a\sqrt{b} + c\sqrt{b} = (a + c)\sqrt{b}$$

$$(2) \quad 5\sqrt{2} + 4\sqrt{2} = (5 + 4)\sqrt{2} = 9\sqrt{2}$$

$$(3) \quad 6\sqrt{7} + 2\sqrt{7} = (6 + 2)\sqrt{7} = 8\sqrt{7}$$

$$(4) \quad 3\sqrt{5} + 2\sqrt{5} = (3 + 2)\sqrt{5} = 5\sqrt{5}$$

$$(5) \quad 2\sqrt{7} + 3\sqrt{7} = (2 + 3)\sqrt{7} = 5\sqrt{7}$$

$$(6) \quad 2\sqrt{3} + \sqrt{3} = 2\sqrt{3} + 1\sqrt{3} = (2 + 1)\sqrt{3} = 3\sqrt{3}$$

$$(7) \quad 5\sqrt{2} + \sqrt{2} = 5\sqrt{2} + 1\sqrt{2} = (5 + 1)\sqrt{2} = 6\sqrt{2}$$

$$(8) \quad 2\sqrt{5} + 4\sqrt{5} + 3\sqrt{5} = (2 + 4 + 3)\sqrt{5} = 9\sqrt{5}$$

$$(9) \quad 5\sqrt{2} + 2\sqrt{2} + 4\sqrt{2} = (5 + 2 + 4)\sqrt{2} = 11\sqrt{2}$$

$$(10) \quad \sqrt{3} + 4\sqrt{3} + 6\sqrt{3} = 1\sqrt{3} + 4\sqrt{3} + 6\sqrt{3} = (1 + 4 + 6)\sqrt{3} = 11\sqrt{3}$$

$$(11) \quad 2\sqrt{3} + 4\sqrt{2} = 2\sqrt{3} + 4\sqrt{2}$$

$$(12) \quad 5\sqrt{7} + 2\sqrt{6} = 5\sqrt{7} + 2\sqrt{6}$$

$$(13) \quad 4\sqrt{2} + 3\sqrt{5} = 4\sqrt{2} + 3\sqrt{5}$$

$$(14) \quad 5\sqrt{2} + 3\sqrt{3} + 2\sqrt{7} = 5\sqrt{2} + 3\sqrt{3} + 2\sqrt{7}$$

Simplify each of the following:

$$(Q1) \quad 2 + 3\sqrt{5} + 6 + 4\sqrt{5}$$

Solution

$$\begin{aligned} 2 + 3\sqrt{5} + 6 + 4\sqrt{5} &= 2 + 6 + 3\sqrt{5} + 4\sqrt{5} \\ &= 8 + (3 + 4)\sqrt{5} = 8 + 7\sqrt{5} \end{aligned}$$

$$(Q2) \quad 6\sqrt{7} + 1 + 4\sqrt{7} + 3 + 2\sqrt{7}$$

Solution

$$\begin{aligned} 6\sqrt{7} + 1 + 4\sqrt{7} + 3 + 2\sqrt{7} &= 6\sqrt{7} + 4\sqrt{7} + 2\sqrt{7} + 1 + 3 \\ &= (6 + 4 + 2)\sqrt{7} + 4 = 12\sqrt{7} + 4 \end{aligned}$$

$$(Q3) \quad 2\sqrt{3} + 5\sqrt{7} + 5\sqrt{3} + 2$$

Solution

$$\begin{aligned} 2\sqrt{3} + 5\sqrt{7} + 5\sqrt{3} + 2 &= 2\sqrt{3} + 5\sqrt{3} + 5\sqrt{7} + 2 \\ &= (2 + 5)\sqrt{3} + 5\sqrt{7} + 2 = 7\sqrt{3} + 5\sqrt{7} + 2 \end{aligned}$$

$$(Q4) \quad 2 + 5\sqrt{2} + \sqrt{3} + 6\sqrt{2} + 7\sqrt{3} + 6$$

Solution

$$\begin{aligned} 2 + 5\sqrt{2} + \sqrt{3} + 6\sqrt{2} + 7\sqrt{3} + 6 &= 2 + 5\sqrt{2} + 1\sqrt{3} + 6\sqrt{2} + 7\sqrt{3} + 6 \\ &= 2 + 6 + 5\sqrt{2} + 6\sqrt{2} + 1\sqrt{3} + 7\sqrt{3} \\ &= 8 + (5 + 6)\sqrt{2} + (1 + 7)\sqrt{3} \\ &= 8 + 11\sqrt{2} + 8\sqrt{3} \end{aligned}$$

$$(Q5) \quad 5^{1/2} + 3\sqrt{2} + 6\sqrt{2} + 7\sqrt{3} + \frac{1}{2}$$

Solution

$$\begin{aligned} 5^{1/2} + 3\sqrt{2} + 6\sqrt{2} + 7\sqrt{3} + \frac{1}{2} &= 5^{1/2} + \frac{1}{2} + 3\sqrt{2} + 6\sqrt{2} + 7\sqrt{3} \\ &= 6 + (3 + 6)\sqrt{2} + 7\sqrt{3} = 6 + 9\sqrt{2} + 7\sqrt{3} \end{aligned}$$

$$(Q6) \quad 2\sqrt{8} + 3\sqrt{3} + 1$$

Solution

$$\begin{aligned} 2\sqrt{8} + 3\sqrt{3} + 1 &= 2\sqrt{4 \times 2} + 3\sqrt{3} + 1 \\ &= 2 \times \sqrt{4} \times \sqrt{2} + 3\sqrt{3} + 1 \\ &= 2 \times 2 \times \sqrt{2} + 3\sqrt{3} + 1 = 4\sqrt{2} + 3\sqrt{3} + 1 \end{aligned}$$

$$(Q7) \quad 3\sqrt{2} + 2 + 2\sqrt{8} + 4\sqrt{2} + 6$$

Solution

$$\begin{aligned} 3\sqrt{2} + 2 + 2\sqrt{8} + 4\sqrt{2} + 6 &= 3\sqrt{2} + 2 + 2\sqrt{4 \times 2} + 4\sqrt{2} + 6 \\ &= 3\sqrt{2} + 2 + 2 \times \sqrt{4} \times \sqrt{2} + 4\sqrt{2} + 6 \\ &= 3\sqrt{2} + 2 + 2 \times 2 \times \sqrt{2} + 4\sqrt{2} + 6 \\ &= 3\sqrt{2} + 4\sqrt{2} + 4\sqrt{2} + 6 + 2 \\ &= (3 + 4 + 4)\sqrt{2} + 8 \\ &= 11\sqrt{2} + 8 \end{aligned}$$

$$(Q8) \quad 5 + 3\sqrt{27} + 2 + 6\sqrt{3} + 2\sqrt{2} + \sqrt{12} + 1$$

Solution

$$\begin{aligned} & 5+3\sqrt{27}+2+6\sqrt{3}+2\sqrt{2}+\sqrt{12}+1 \\ &= 5+3\sqrt{27}+2+6\sqrt{3}+2\sqrt{2}+\sqrt{12}+1 \\ &= 5+3\times\sqrt{9\times 3}+2+6\sqrt{3}+2\sqrt{2}+\sqrt{4\times 3}+1 \\ &= 5+3\times 3\times\sqrt{3}+2+6\sqrt{3}+2\sqrt{2}+2\times\sqrt{3}+1 \\ &= 5+9\sqrt{3}+2+6\sqrt{3}+2\sqrt{2}+2\sqrt{3}+1 \\ &= 5+2+1+9\sqrt{3}+6\sqrt{3}+2\sqrt{3}+2\sqrt{2} \\ &= 8+(9+6+2)\sqrt{3}+2\sqrt{2} \\ &= 8+17\sqrt{3}+2\sqrt{2} \end{aligned}$$